S282 Astronomy

Variable stars

Study time: 2 hours

Summary

In this spreadsheet activity you will analyse some results of observations of Cepheid variable stars. You will first investigate the Cepheid period–luminosity relation using data from stars in the Small Magellanic Cloud. (*Note there is an error in the summary on the multimedia guide page, which incorrectly says 'Large Magellanic Cloud'*). Then you will determine the period of an unknown star by plotting a light curve of a month's observations. This period will be used to determine the luminosity and hence an estimate of the distance to the star.

It is recommended that you attempt this activity after you have read to the end of Chapter 3 of *An Introduction to the Sun and Stars* and after you have completed the spreadsheet activities 'Sunspot number' and 'Stellar distance and motion'.

Learning outcomes

- Develop your understanding of how Cepheid variable stars are used to determine distances.
- Develop your spreadsheet skills, particularly with the use of formulae and graph plotting and interpretation.
- Use a spreadsheet to determine the period of a variable star when the light curve is randomly sampled.

Background to the activity

Cepheids are pulsating variable stars. They exhibit regular changes in luminosity, temperature and radius due to instabilities in the outer layers of the stars. They are named after the first to be discovered, δ Cephei, in 1784, but it would be over a century before their importance as standard candles, (objects with known luminosity that can be used to determine astronomical distances) was recognized.

Henrietta Leavitt (Figure 3.32 in *An Introduction to the Sun and Stars*) examined hundreds of photographic plates obtained between 1893 and 1906 at Harvard College's observatory in Peru to produce a catalogue of 1777 variable stars in the Magellanic Clouds (two regions that look like detached parts of the Milky Way, visible in the southern hemisphere that are now known to be small external galaxies). A total of 16 stars appeared in enough plates for their periods to be determined, and at this stage she noticed that the brighter ones appeared to have the longest period.

By 1912 she had obtained periods and magnitudes for 25 stars in the Small Magellanic Cloud (SMC) which confirmed this period–luminosity relation. She recognized the characteristic shape of the light curves and wrote:

> Since the variables are probably at nearly the same distance from the Earth, their periods are apparently associated with their actual emission of light, as determined by their mass, density and surface brightness … It is to be hoped, also, that the parallaxes of some variables of this type may be measured.

This only hinted at the great importance that Cepheids would attain in the determination of distances to external galaxies.

In the first part of this exercise you will use data for the 25 stars measured by Leavitt to plot the apparent magnitude *m* versus log (period) and obtain a best fit straight line of the form

$$m = c + b \log P \tag{1}$$

where *b* is the slope of the line and *c* is the intercept. This graph, plotted by Leavitt, led to her confirmation of the period–luminosity relation for the SMC variables. Without any independent measurement of the distance of the SMC it was not possible at the time to calibrate this relationship in terms of the intrinsic brightness of the stars. Since the SMC stars are all at the same distance then we know from Equation 3.16 in *An Introduction to the Sun and Stars* that

$$M = m - 5 \log d + 5 \tag{2}$$

(where *d* is in parsecs) tells us that $M = m$ + a constant and so

$$M = a + b \log P \tag{3}$$

where the slope, *b*, is the same as in Equation 1 and the new constant, *a*, provides the calibration of the period–luminosity relation.

Danish astronomer Ejnar Hertzsprung (Figure 4.2a in *An Introduction to the Sun and Stars*) realized that if the period–luminosity relation could be calibrated, (i.e. the value of the constant *a* determined) then the absolute magnitudes, *M*, of Cepheids could be determined directly from their periods. Comparison with their measured apparent magnitudes, *m*, would then yield their distances, *d*, using Equation 2.

The actual determination of the zero point of the calibration was a difficult process as no Cepheids were close enough for parallax to be determined. Even today there is some uncertainty in the value of *a*.

This activity is divided into two parts. If you don't want to complete the activity in one go then you can save your data and return at any point. However it is recommended that you complete each part in one sitting if possible so you don't lose track of what you are doing.

You will be using data provided in an existing spreadsheet. You should now be familiar with the use of spreadsheets. For a reminder of the basic functions or for more information, refer to the *Using Spreadsheets* guide on the course website.

The instructions given here assume that you will be using the StarOffice™ package that is supplied on the OU Online Applications CD-ROM. If you are already familiar with using another spreadsheet package (such as Microsoft Excel) you may want to use that to carry out the activity (we have also supplied the required data file in Excel format). However, before starting, you should be aware that these notes only give instructions on how to manipulate the StarOffice spreadsheet.

Part 1 Cepheids in the Small Magellanic Cloud

Open the raw data file

Before you start it would be a good idea to set up a folder in which you can store the results of your work.

The raw data for this activity is contained in a file called 'S282 Variable star.sxc'. (The Excel version of the file is called 'S282 Variable star.xls').

- Start the S282 Multimedia guide program, open the folder called 'Stars' then click on the icon for this activity ('Variable stars').
- Press the Start button to access the folder on the DVD containing StarOffice and Excel versions of the file containing the raw data.
- Open the file you wish to use by double-clicking on it.

Save a copy of the file

Before you can make changes to the file, you must save it to your hard disk.

- Use the File | Save as... menu command to save a copy of the spreadsheet into your work folder.

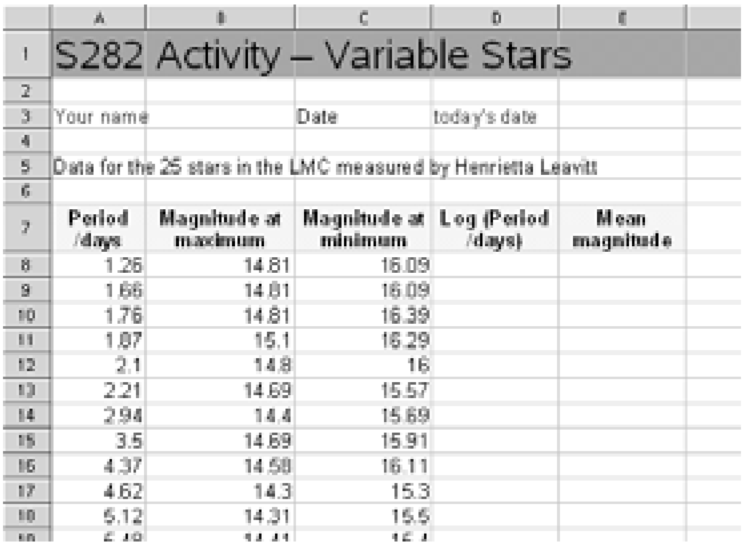
As you make changes to the spreadsheet you should save your work regularly to prevent any changes from being lost. From time to time make a backup copy of your work (using a different filename) in case you need to go back to an earlier stage.

Preparing the data

The spreadsheet contains two worksheets, labelled Cepheid light curve and PL relation on the tabs at the bottom of the spreadsheet.

- Select the PL relation worksheet by clicking on the tab. You should see the table shown in Figure 1 (*overleaf*).

The table contains apparent magnitudes for the maximum and minimum brightness and the pulsation period in days for the 25 Cepheids identified by Henrietta Leavitt in the Small Magellanic Cloud. (*You should change the label 'LMC' to 'SMC' here.*) In order to determine the slope of the period–luminosity relation for these stars you will need to plot the mean apparent magnitude against the logarithm of the period. The column headings are already prepared for this.



	A	B	C	D	E
1	S282 Activity – Variable Stars				
2					
3	Your name		Date	today's date	
4					
5	Data for the 25 stars in the LMC measured by Henrietta Leavitt				
6					
7	Period /days	Magnitude at maximum	Magnitude at minimum	Log (Period /days)	Mean magnitude
8	1.26	14.81	16.09		
9	1.66	14.81	16.09		
10	1.76	14.81	16.39		
11	1.87	15.1	16.29		
12	2.1	14.8	16		
13	2.21	14.69	15.57		
14	2.94	14.4	15.69		
15	3.5	14.69	15.91		
16	4.37	14.58	16.11		
17	4.62	14.3	15.3		
18	5.12	14.31	15.5		

Figure 1 The PL relation worksheet prior to carrying out any calculations. (Note there is a typographical error in the spreadsheet – 'LMC' should be 'SMC'.)

- In cell D8 type the formula `=log10(A8)` and press Enter or click the symbol to determine the logarithm (to the base 10 – see Box 3.2 of *An Introduction to the Sun and Stars*) of the period in column A8. You can then apply this formula to each cell in the column by *dragging* (as described below).
- Select cell D8, which contains the formula, click on the little black square in the bottom right-hand corner of the cell and drag down to row 33, keeping the mouse button held down.
- This will replicate the formula all the way down the column, updating the row numbers in the formula. You should now have a column containing log(period) values for all the stars.

(At this point you may want to reformat the column to show more decimal places – refer to *Using Spreadsheets* if you cannot remember how to do this.)

Column E is labelled Mean magnitude.

Question 1

What formula do you need to type in cell E8 to calculate the mean apparent magnitude?

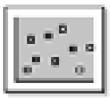
- Apply this formula to the remaining cells in the column as before.

Create the chart

The data in columns D and E can now be used to plot a chart of magnitude versus log(period). If you have completed the 'Sunspot number' activity you will already be familiar with plotting graphs in a spreadsheet. The instructions are repeated here:

- Highlight both columns by clicking on cell D8 and dragging all the way down and across to cell E33.
- From the main menu, select Insert | Chart ....

- A box labelled AutoFormat Chart will appear. You don't need to change anything here, so select Next>> to move onto the next step.
- From the Choose a chart type options, select XY Chart, i.e.



- followed by Next>> to move onto the next step.
- From the Choose a variant options, select the same icon (Symbols only). If you prefer to have gridlines on both the X and Y axes, or neither, you can select the appropriate boxes. Select Next>> to move onto the next step. (Note that while going through these steps, you can always press Back to go to an earlier step if you need to correct something.)
- In AutoFormat Chart make the following changes:

  The tick-box next to Legend should be cleared (i.e. without a tick).

  Make sure the X Axis and Y Axis tick-boxes are ticked – this will cause the axis titles to be displayed.

  Insert the text for titles as follows:

Chart title:	SMC Cepheids (*Note*: LMC should be SMC)
Axis title / X Axis:	Log(Period/days)
Axis title / Y Axis:	Apparent magnitude

- Finally, press the Create button – this will draw the chart.

Formatting the chart

Having created the chart you will need to format it to improve the way in which it displays these data, and maybe tidy it up to make it visually more appealing. In order to make changes, the chart must be *selected*: there are two different levels of selection, signified by different borders around the chart:

- To resize, move or delete the chart: single click; the chart has green selection handles.
- To edit or format the chart: double-click; the chart has grey border and small black squares.
- To deselect the chart: click on any other cell in the spreadsheet.

Note that to swap between the 'resize, move, delete' selection and the 'edit, format' selection it is necessary to deselect the chart, before selecting it again.

The first thing that you are likely to want to do is to resize the chart to make it larger. To do this select it by single clicking and then dragging one of the green selection handles as required.

To change other aspects of the appearance of the chart you will need to make sure that the chart is selected for formatting (grey border and black squares). You can change the formatting of just about any element of the graph by double-clicking on the part you want to change. As you move the mouse pointer over the chart a small label will appear telling you which item will be modified (you may need to be quite precise with the mouse to select the item you want!). In this way you can change colours, add or remove background shading, gridlines and so on.

There are many different things that you can do to change the appearance of your chart. Some of them will be cosmetic, i.e. to make the chart more visually appealing, or easier to use. There is, however, one aspect of this chart that does

not satisfy the normal astronomical convention for plotting magnitudes. The magnitude scale has the brightest objects with the most negative magnitudes. Spreadsheet plotting routines automatically show the axes with the largest numerical value at the top. Although some spreadsheets (such as Microsoft Excel) allow the axes to be reversed, this facility is not present in StarOffice. It is not important for this part of the activity, but you should remember that the brightest stars appear at the *bottom* of the figure.

If you want to re-apply any of the chart type selections or titles choose Format | AutoFormat... from the main menu; this will take you through the AutoFormat Chart steps again.

Changes you may want to make are removing the grey background (double-click on chart area) adjusting the axes (click near the axes or use Format | Axis | X Axis or Format | Axis | Y Axis) and adding a border around the chart and plotting area (use Format | Chart area and Format | Chart wall respectively). Experiment with the various formatting options until you are happy with the appearance of your graph.

Question 2

Comment on the relationship between log(period) and apparent magnitude for the 25 stars plotted on your graph.

The period–luminosity relation

You can now use your graph to determine how well the data is represented by Equation 1:

$$m = c + b \log P$$

Question 3

What do the constants *b* and *c* represent on your graph?

The spreadsheet contains software that automatically determines the best fit straight line to a set of data as follows:

- Ensure the chart is selected then select Insert | Statistics from the main menu. (Alternatively you can double-click on the data points to reveal the Data Series screen and select the Statistics tab.) The box shown in Figure 2 (*overleaf*) will appear.
- In the Regression Curves box select the Linear regression type fit from the different types of line fitting available.
- Click on OK and the best fit line will appear.

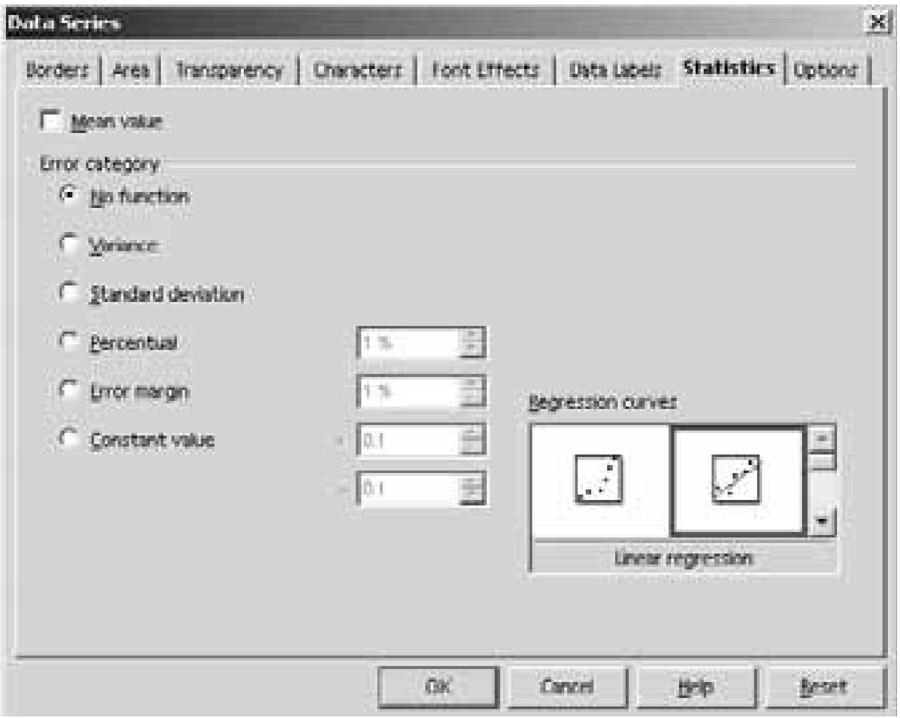


Figure 2 Window for adding straight line fit to data on a graph.

Question 4

The magnitudes of the stars do not perfectly fit the straight line. Assuming the period–luminosity relation can be represented perfectly by a straight line on this type of plot can you explain why you would not expect the stars to fit it exactly?

Question 5

The distance of the SMC is now known to be approximately 55 kpc with a diameter of about 3 kpc. Use Equation 2 to show that this is compatible with the data and your answer to Question 4.

In fact, the period–luminosity relation plotted for many Cepheids will show scatter for a number of other reasons including:

(i) The absorption of light by interstellar dust (see Section 4.3.3 in *An Introduction to the Sun and Stars*) will affect the observed apparent magnitudes and must be corrected for.

(ii) The structure of a star depends primarily on its mass, but also its composition and energy source (Chapter 6 of *An Introduction to the Sun and Stars*). Although Cepheids are all at a similar stage of their evolution, with the same nuclear energy source (as you will see in Chapter 7 of *An Introduction to the Sun and Stars*) they may have different *initial* compositions and therefore slightly different structures and pulsation periods for a given mass (this effect is described in Chapter 9 of *An Introduction to the Sun and Stars*). This effect is sufficiently important that two classes of Cepheids exist with two different period–luminosity relations. We will not consider this further in this activity. If no distinction is made then we assume that the Cepheids formed in regions similar to that in which the Sun formed (and should correctly be called *classical Cepheids* or *Type 1 Cepheids*).

The 25 stars you have examined in the SMC do not provide a sufficiently large number of stars to represent the whole of the Cepheid population, and hence do not allow an accurate determination of the period–luminosity relation. They were however, as you have seen, sufficient for Henrietta Leavitt to recognize that the relationship existed. The exact details of the period–luminosity relation are being continuously refined as improved observational data become available. For example the Hipparcos mission (see Section 3.2.2 of *An Introduction to the Sun and Stars*) provided parallaxes for stars at greater distances and significant strides were made in determining astronomical distances.

For the next part of the activity we will use a recent evaluation of the period–luminosity relation:

$$M_V = -2.8 \log P - (1.4 \pm 0.2) \tag{4}$$

where P is the period in days.

Part 2 The light curve and distance of an unknown Cepheid

In this part of the activity you will analyse some observed data for an unknown Cepheid to determine its pulsation period and hence estimate its absolute magnitude and distance.

- Open the second worksheet in the spreadsheet, which is labelled ‘Cepheid light curve’. You should see the table shown in Figure 3:

	A	B	C	D	E	F	G	H	I
1	S282 Activity – Variable Stars								
2									
3	Your name		Date	today's date					
4									
5	Observations of Variable Star 'X'								
6						Test Period =			days
7									
8									
9	Night of observation	Time	Apparent visual magnitude	+/-	Time in days	Integer days from Jan 4	Time from Jan 4.0 /days	Phase	Folded phase
10	2003 Jan 4/5	18:58	9.11	0.04					
11		0:15	8.93	0.04					
12	2003 Jan 5/6	19:02	8.48	0.04					
13		19:46	8.51	0.04					
14		20:52	8.58	0.06					
15		21:03	8.42	0.07					
16		2:42	8.45	0.04					
17		4:20	8.59	0.08					
18	2003 Jan 6/7	22:35	8.57	0.04					
19	2003 Jan 7/8	20:28	8.76	0.04					
20		0:51	8.8	0.04					
21		1:13	8.88	0.06					

Figure 3 The Cepheid light curve worksheet prior to carrying out any calculations.

The star has been observed for a period of a month. The date, time, magnitude and estimated uncertainty in the magnitude are provided.

Folded light curves

The period of pulsation and therefore change in brightness is unknown but if the star is a Cepheid it will be in the range of 1 to 100 days. Even if the period is very short then up to half the light curve will be visible in one night. If the period is as long as 100 days then the magnitude will be virtually constant over the course of

one night but will gradually change from night to night. If the period is a few days then the magnitude will appear to change randomly from night to night. Although it will be possible to distinguish between these cases it is highly unlikely that you will be able to determine the period simply by looking at the light curve.

When magnitudes are measured for many small parts of a light curve, which may span a number of pulsation cycles it is convenient to plot what is called a *folded* light curve. Instead of a plot of time in hours or days versus magnitude, the time, t, is divided by the period, P, to produce the *phase* (t/P). The phase therefore consists of two parts: the whole number, which tells you the number of periods that have passed since time $t = 0$, and the decimal part, which tells you what part of the cycle you have reached. The light curve can then be plotted as the decimal part of the phase versus magnitude. This has the advantage of overlaying all the data for many pulsations into one period so that details in the structure of the light curve can be seen.

Of course, you do not initially know the period, but this light curve can be used to determine it as you will see.

First, however, you need to convert each date and time into a phase.

Preparing the data

Columns A and B contain the date and time of each observation. In order to perform calculations these need to be converted into a single number. It is convenient to choose a fixed date and time as zero and to then calculate all other times in units of hours or days from this point. The start of the first day of observation is the most sensible to use so 4 Jan 2003 at 0:00 can be set as $t = 0$ and time measured in days from this point.

Column B contains the time in the format hh:mm which is recognized by StarOffice as a time. If it is used in a calculation it will automatically be converted into decimal *days*. Unfortunately the date in column A is not written in a convenient form that the spreadsheet recognizes as a date so we will need to determine the time t in stages. (You can see which formats are recognized by looking at the options in Format | Cells and selecting date.)

The value for cell E10, which is the time (cell B10) in days, is therefore the trivial formula `=B10`. Apply this to the whole column.

Question 6

How many decimal places are appropriate for the data in column E?

Adjust the formatting of the column so that the data appear as numbers with the appropriate number of decimal places.

Column F is the number of whole days from 4 Jan 2003. Enter these values manually by inspection of column A. You need to take great care here because:

(i) the day number changes at midnight so that cell F10 contains the value 0, F11 to F15 contain 1, etc.)

(ii) there are some days when no data have been taken (presumably because of bad weather) and there are therefore gaps in the day number.

As a check that you have performed this correctly, cell F55 should contain the value 12 and cell F76 should contain the value 23.

Column G contains the total time in days from Jan 4.0.

Question 7

What is the formula to be written in cell G10 and what is the value?

- Apply this to the whole column by dragging and reformatting to show the appropriate number of decimal places.

Column H is where you will calculate the phase. This is calculated using the (currently unknown) period. The yellow box above the table contains the value of the period in cell H6 (Figure 4) in days. Enter an initial guess (say 1) here.

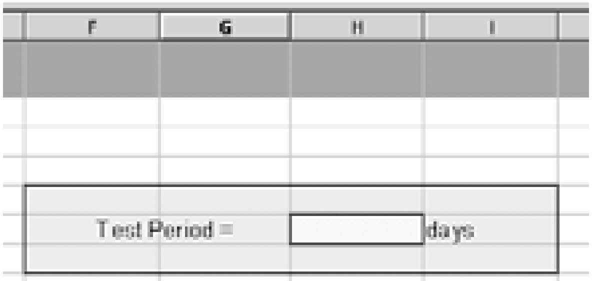


Figure 4 Location in the spreadsheet where the pulsation period is entered.

- Enter the formula for the phase, `=G10/$H$6`, in cell H10.

The $ symbols before the ‘H’ and the ‘6’ in the cell address for the period signify that they will not be updated when the formula is dragged. This is called an absolute cell reference. (Note if the cell shows `Err.503` then an attempt to divide by zero has been made, i.e. you have not entered a value in the period box, cell H6.)

- Apply this formula to the whole column by dragging.
- Inspect the entry in cell H11. You can see it should read `=G11/$H$6` as required. If the $ symbols had not been entered it would read `=G11/H7`, which would give an erroneous answer.
- Reformat the column to the appropriate number of decimal places.

You now have values of phase for each date and time. However, this is not sufficient to plot a folded light curve. You need to remove the integer (whole number) part of the phase so that all periods are plotted on top of each other. This is achieved by entering the formula `=H10-INT(H10)` in cell I10. Apply this to the whole column by dragging and reformatting to the appropriate number of decimal places.

You now have all the data ready to plot the folded light curve.

Plotting the folded light curve

The procedure for plotting as described above for the magnitude versus period graph can be repeated with a few changes. Before plotting the light curve there are three factors to consider.

First, you will want to plot the magnitude on the vertical axis (Y axis) and folded phase on the horizontal axis (X axis). The procedure used in Part 1 of this activity requires data in adjacent columns with the X axis data first. It is possible to select non-adjacent columns and correct for data being in the wrong order, but a simpler solution is to copy the magnitude column next to the folded phase column. Before you do this you need to consider the following point.

Second, because of the limitations of StarOffice, the light curve will appear upside down. This is now important as you will want to identify the characteristic pattern for a Cepheid light curve and have the brightest magnitudes at the top. It therefore makes sense to multiply the magnitudes by –1 to reverse the scale and remember to ignore the negative sign in the plot:

- Enter the formula `=–C10` in cell J10 and apply to the column by dragging. Adjust the formatting and label the column '–Magnitude for plotting'.

Third, the folded light curve will show data from phase 0.0 to 1.0. This means that there will be one maximum and one minimum visible on the light curve. In order to see the pattern of the light curve more easily and judge when the shape is most consistent it is convenient to plot two periods (i.e. repeat the data between phase 1.0 and 2.0. This means that wherever the maximum or minimum lies there will be one complete cycle visible.

- Select cell I78 and enter the formula `=I10+1`. Apply this formula to the entire data set by dragging down to cell I144. The cells I78 to I144 will now contain the folded phase values between 1 and 2. Reformat if required.
- Next you will need to copy the magnitude data which apply to these folded phases. Select cell J78 and enter the formula `=J10`. Apply this formula to the entire data set by dragging down to cell J144.

The data in columns I and J can now be used to plot a chart of magnitude versus folded phase.

- Highlight both columns by clicking on cell I10 and dragging all the way down and across to cell J144.
- From the main menu select Insert | Chart ...
- Select Next>> in the AutoFormat Chart.
- From the Choose a chart type options select XY Chart, followed by Next>>.
- From the Choose a variant options select Symbols Only, followed by Next>>.
- In AutoFormat Chart clear the tick-box next to Legend, and make sure the X Axis and Y Axis tick-boxes are ticked.

  Insert the titles:

Chart title:	Light curve for star X
Axis title / X Axis:	Phase
Axis title / Y Axis:	–Apparent visual magnitude

- Finally, press the Create button to draw the chart. (You will probably see an apparent error message stating that the X-axis values must be sorted. It is not necessary so click No.) The chart will now appear near the top of your worksheet (you will have to scroll back to see it).
- Reformat the chart as desired.

At this stage the data will appear as a scatter (Figure 5) since the correct period has not yet been found.

There is one further factor that should make it easier to judge the best fit period. Each observed magnitude has an associated uncertainty listed in column D. It is best to add these uncertainties to the plotted light curve as error bars.

Unfortunately the StarOffice spreadsheet does not allow *different* error bars to be plotted for each point so it is necessary to plot a *typical* value for each point.

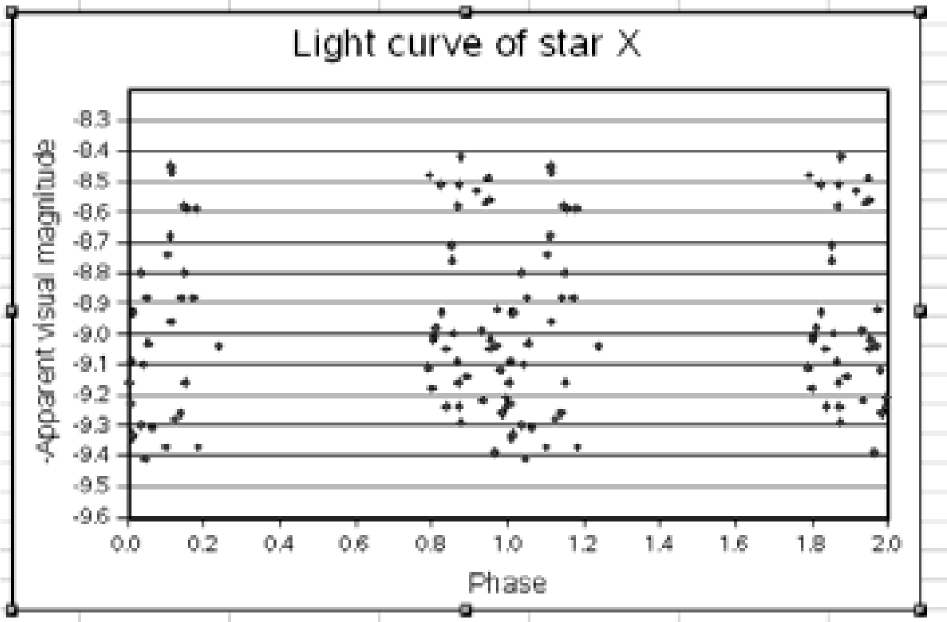


Figure 5 Light curve after initial plotting and formatting.

Question 8

Using the spreadsheet data, what is a representative value for the typical uncertainty in an observed apparent visual magnitude? Can you think of a reason why all the values which differ from this one are larger?

In StarOffice error bars are usually added using Insert | Statistics from the main menu. (Alternatively you can double click on the data points to reveal the Data Series screen and select the Statistics tab) as shown in Figure 2. If Constant value is selected from the Error category choices then the uncertainties can be entered in the + and – boxes. However, if you try this you will discover that StarOffice will not allow you to enter values smaller than 0.1. There is another way around this problem that involves changing the symbols used in the graph plotting, as follows:

- Double-click to select the graph for editing.
- Place the cursor over one of the data points on the graph and double click. A Data Series window will open.
- Select the Borders tab and the screen shown in Figure 6 should appear.

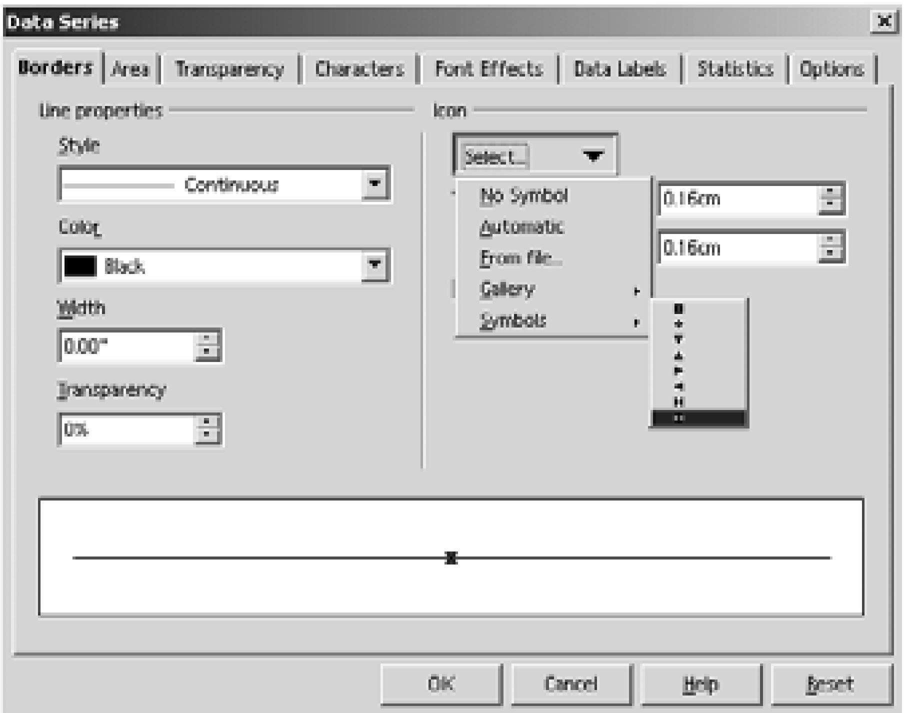


Figure 6 Window for formatting the data symbols plotted on a graph.

- You can adjust the symbol shape using the Select button and taking the Symbols option. Choose the bottom symbol (which you will see resembles an error bar) to replace the default diamond shaped symbol (as shown in Figure 6).
- De-select the Keep ratio box and adjust the symbol size using the Width and Height boxes; 0.1 cm is a reasonable value for the Width. Try 0.4 cm as a first attempt at the Height.
- Click the OK box to view the result. You will need to experiment with the Height to obtain the best value to match plus and minus the uncertainty from Question 8 on your particular plot. (This is rather fiddly!)

The light curve should now look like Figure 7.

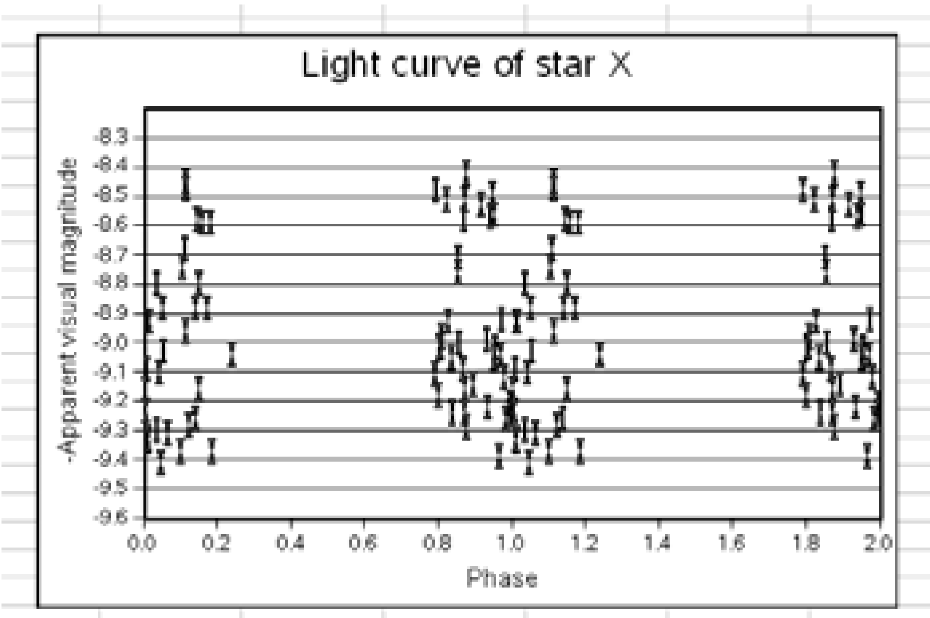


Figure 7 The folded light curve before searching for the best period.

Determining the period of star X

You can now experiment with entering new values for the period in cell H6. Examine the appearance of the folded light curve until you think you have the best representation of a smooth light curve. You will need to confirm that it really is a Cepheid variable (it will not surprise you to learn that it is!) based on the light curve shape (see Figure 3.31 in *An Introduction to the Sun and Stars* if you need reminding).

Note that the uncertainties listed in column D represent the estimated uncertainties in the data so they give an indication of how much scatter you might expect. The points with the larger uncertainties may well lie further from the general trend of the light curve. When examining the light curve, you can check if points that do not fit the trend well correspond to these less reliable data. A more sophisticated spreadsheet package would allow the actual uncertainties to be plotted making this process easier.

Question 9

What is your estimate of the pulsation period of the Cepheid, star X? What is your estimate of the uncertainty in this value?

Estimating the distance of the Cepheid

Using the period–absolute magnitude relation as given in Equation 4:

$$M_V = -2.8 \log P - (1.4 \pm 0.2)$$

you can derive the absolute visual magnitude of star X. The uncertainty in the absolute visual magnitude will be a combination of the uncertainty in the relation itself (0.2) and that resulting from the uncertainty in the period.

Question 10

What is your derived value of the absolute visual magnitude of star X and your calculated uncertainty in this value?

(Hint: derive the maximum and minimum likely values of M_V using $P \pm \Delta P$ in Equation 4 without the ± 0.2. Then combine the resultant uncertainty with the ± 0.2. See previous spreadsheet activities if you are still unsure how to do this.)

Question 11

What is your estimate of the mean apparent visual magnitude of star X from your light curve? What is your estimated uncertainty in this value?

Question 12

Use Equation 2 to determine the distance of star X and the uncertainty in this value. (Hint: to derive the uncertainty in d, derive the uncertainty in $(m_V - M_V)$ and then determine the value of d for the maximum and minimum values.)

Question 13

What assumption has been made in deriving this distance?

Answers to questions

Question 1

The formula for mean magnitude should be `=(B8+C8)/2`.

Question 2

The data do show a trend with the longest period stars having the brightest magnitudes. It is approximately a straight line but there is some scatter in the data.

Question 3

The constant b is the gradient of the best fit line on your graph. The constant c is the intercept with the Y axis, i.e. the apparent magnitude for a value of log(period/days) = 0, which corresponds to the apparent magnitude for a star with a period $P = 1$ day.

Question 4

Your graph should look similar to Figure 8.

There are two reasons why the stars would not be expected to lie exactly on the best fit line:

1 There will be observational errors present in the data (although we have not been supplied with an estimate of the uncertainties in the data to check whether they are consistent with the scatter).

2 Although we have assumed all the stars are at the same distance (so that the apparent magnitudes differ from the absolute magnitudes by exactly the same amount), the SMC does have a finite size. Any star might lie at the front or the back of the SMC as seen from the Earth and therefore lie slightly closer (or further away) than the average. This will make its apparent magnitude slightly smaller (or larger) than it would if it lay at the average distance.

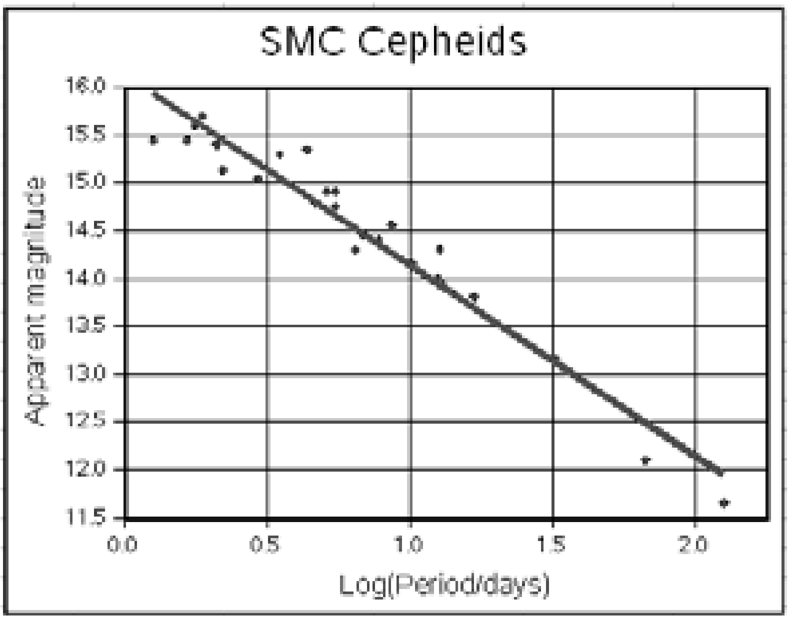


Figure 8 The relationship between apparent magnitude and log(period) for the SMC Cepheids observed by Henrietta Leavitt.

Question 5

If the diameter of the SMC is not negligible compared with the distance from the cloud to the Earth then we can calculate the effect this would have on the apparent magnitude of a star.

Using Equation 2:

$$M = m - 5 \log d + 5$$

The absolute magnitude M of a star is a measure of its luminosity. We can work out the effect on the apparent magnitude m, of changing d. Rearranging Equation 2 gives

$$m = M + 5 \log d - 5$$

As the cloud has a diameter of 3 kpc, a star at the front of the cloud will have a distance of 1500 pc *less than* the mean distance of the stars in the cloud and a star at the back of the cloud will have a distance 1500 pc *more than* the mean distance of the stars in the cloud

so $\quad m_{\text{min}} = M + 5 \log(55\,000 - 1500) - 5 = M + 18.64$

and $\quad m_{\text{max}} = M + 5 \log(55\,000 + 1500) - 5 = M + 18.76$

The finite depth of the SMC can therefore introduce a scatter of up to 0.12 magnitudes in the observed apparent visual magnitudes compared to the best fit line.

Your graph shows that this can explain only part of the scatter in the data.

Question 6

The times in column B are quoted to the nearest minute and can therefore be no more accurate than ±0.5 minutes. 0.5 minute = $1/(2 \times 24 \times 60) = 0.000\,35$ days. The time in days should therefore be quoted to no more than 4 decimal places.

Question 7

The formula for cell G10 is `=E10+F10`. The value should be 0.7903.

Question 8

The typical uncertainty, which applies to most of the data, is ±0.04 magnitudes (column D). This is likely to be due to the limitations of the telescope and detector used for the observations under good observing conditions, so cannot be improved upon without a bigger telescope or more sensitive detector. The larger values are likely to be due to external factors, such as poor weather.

Question 9

You should obtain a value close to 7.6 days. Anything between 7.4 and 7.8 looks reasonable so the uncertainty is ±0.2 days. Your folded light curve after selecting the best period should look like Figure 9.

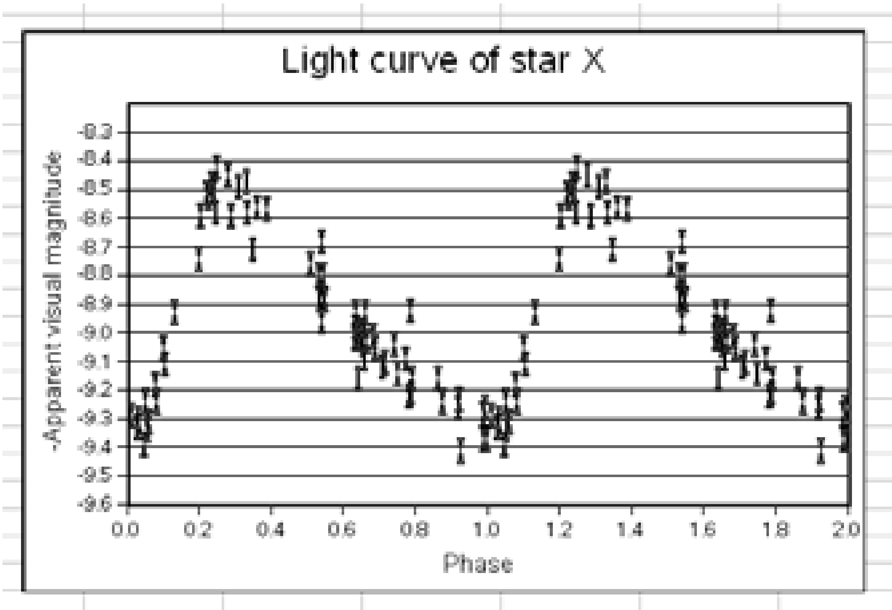


Figure 9 The final folded light curve of star X for a period of 7.6 days.

Question 10

Substituting P = 7.6 days into Equation 4 gives an absolute visual magnitude of

$$M_V = -2.8 \log(7.6) - (1.4) = -3.87$$

Applying the uncertainty in P gives

$$M_{V(max)} = -2.8 \log(7.6 + 0.2) - (1.4) = -3.90$$

$$M_{V(min)} = -2.8 \log(7.6 - 0.2) - (1.4) = -3.83$$

So the uncertainty in the magnitude resulting from the period uncertainty is approximately ±0.04.

Combining this with the uncertainty in the period–luminosity relation (i.e. ±0.2)

$$\Delta M_V = \sqrt{[(\Delta M_{V(\text{period})})^2 + (\Delta M_{V(\text{PL relation})})^2]}$$

$$= \sqrt{[(0.04)^2 + (0.2)^2} = 0.20$$

(The uncertainty resulting from the period is negligible compared with the scatter in the period–luminosity relation.)

The absolute visual magnitude of star X is therefore $M_V = -3.87 \pm 0.20$.

(Note that if this were the final answer you would probably quote it as $M_V = -3.9 \pm 0.2$. However, it is best to retain an extra decimal place during the calculations to prevent rounding errors. The final answer, in this case the distance, can then be given to the appropriate number of significant figures.)

Question 11

The approximate maxima and minima of the light curve are 8.45 and 9.35. The mean apparent visual magnitude is therefore 8.9 with an estimated uncertainty of ±0.05 based on the scatter in the points.

Question 12

Rearranging Equation 2, $M_V = m_V - 5 \log d + 5$, gives

$$d = 10^{(m_V - M_V + 5)/5}$$

$$d = 10^{(8.9+3.87+5)/5} = 3581 \text{ pc}$$

The uncertainty in the distance is a result of the uncertainty in $m_V - M_V$. The uncertainty in this quantity is

$$\sqrt{[(0.05)^2 + (0.20)^2]} = 0.21$$

$$m_V - M_V = 12.77 \pm 0.21$$

The resultant maximum and minimum likely values of d are therefore:

$$d_{max} = 10^{(12.77+0.21+5)/5} = 3945 \text{ pc}$$

$$d_{min} = 10^{(12.77-0.21+5)/5} = 3251 \text{ pc}$$

The uncertainty in d is therefore approximately ±400 pc.

The distance to star X is therefore $d = (3600 \pm 400)$ pc.

Question 13

It has been assumed that there is no interstellar absorption.